Taylor series

1. Write down the 1st-order Taylor series for $y(t-2h)$, $y(t-h)$ and $f(x+h)$.

Answer:

$$y\left(t-2h\right)=y\left(t\right)-2y^{(1)}\left(t\right)h+2y^{(2)}\left(\tau\right)h^2$$

$$y\left(t-h\right)=y\left(t\right)-y^{(1)}\left(t\right)h+\frac{1}{2}y^{(2)}\left(\tau\right)h^2$$

$$f\left(x+h\right)=f\left(x\right)+f^{(1)}\left(x\right)h+\frac{1}{2}f^{(2)}\left(\xi\right)h^2$$

2. Write down the 2nd-order Taylor series for $y(t-3h)$ and $f(x+2h)$.

Answer:

$$y\left(t-3h\right)=y\left(t\right)-3y^{(1)}\left(t\right)h+\frac{9}{2}y^{(2)}\left(t\right)h^2-\frac{9}{2}y^{(3)}\left(\tau\right)h^3$$

$$f\left(x+2h\right)=f\left(x\right)+2f^{(1)}\left(x\right)h+2f^{(2)}\left(x\right)h^2+\frac{4}{3}f^{(3)}\left(\xi\right)h^3$$

3. Write down the 3rd-order Taylor series for $y(t-h)$ and $f(x+3h)$

$$y\left(t-h\right)=y\left(t\right)-y^{(1)}\left(t\right)h+\frac{1}{2}y^{(2)}\left(t\right)h^2-\frac{1}{6}y^{(3)}\left(t\right)h^3+\frac{1}{24}y^{(4)}\left(\tau\right)h^4$$

$$f\left(x+3h\right)=f\left(x\right)+3f^{(1)}\left(x\right)h+\frac{9}{2}f^{(2)}\left(x\right)h^2+\frac{9}{2}f^{(3)}\left(x\right)h^3+\frac{27}{8}f^{(4)}\left(\xi\right)h^4$$

4. Which is correct, and what is an easy way to remember that that is the correct formulation?

$$f\left(t\right)=f\left(t_0\right)+f^{(1)}\left(t_0\right)\left(t-t_0\right)+\frac{1}{2}f^{(2)}\left(x\right)\left(t-t_0\right)^2+\frac{1}{6}f^{(3)}\left(\xi\right)\left(t-t_0\right)^3$$

$$f\left(t\right)=f\left(t_0\right)+f^{(1)}\left(t_0\right)\left(t_0-t\right)+\frac{1}{2}f^{(2)}\left(x\right)\left(t_0-t\right)^2+\frac{1}{6}f^{(3)}\left(\xi\right)\left(t_0-t\right)^3$$

Answer: The first is correct. An easy way this author has to remember this is: if the derivative is positive and $t > t_0$, then $f(t)$ must be greater than $f(t_0)$, and thus $f^{(1)}(t_0)$ must be multiplied by a positive value, and if $t > t_0$, then $t - t_0$ is positive while $t_0 - t$ would be negative.

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